TURLI KO'RINISHDAGI FUNKSIONAL TENGLAMALARNI YECHISH

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Matematika fani o'qituvchisi

1. Agar $f: R\backslash\{0\} \to R$ uchun $f\left(2+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}\ \forall\, x \in R\ \{0\}$ bo'lsa, $f(x)$ funksiyani toping.

Yechim: $t = 2 + \frac{1}{x}$ belgilash kiritamiz, bu yerdan $x = \frac{1}{t-2}$ kelib chiqadi.

O'rniga qo'yib $f(t) = \left(\frac{1}{t-2}\right)^2 + (t-2)^2$ ga ega bo'lamiz.

Demak, $f(x) = \frac{1}{(x-2)^2} + (x-2)^2$ ekanligi kelib chiqadi.

2. Agar $f: R\backslash\{0\} \to R$ uchun $3f(x) + 2f\left(\frac{1}{x}\right) = x\ \forall\, x \in R\ \{0\}$ bo'lsa, $f(x)$ funksiyani toping.

Yechim: x ni $\frac{1}{x}$ bilan almashtirib olamiz, bu yerdan $3f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x}$ kelib chiqadi.

$$\begin{cases} 3f(x) + 2f\left(\frac{1}{x}\right) = x \\ 3f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x} \end{cases}$$

Sistemani ishlab

$$f(x) = \frac{3x^2 - 2}{5x}$$

ekannligini olamiz.

3. Agar $x^2 f(x) + f(1-x) = 2x - x^4\ \forall\, x \in R$ bo'lsa, $f(x)$ funksiyani toping.

Yechim: x ni $1-x$ bilan almashtirib olamiz, bu yerdan

$$(1-x)^2 f(1-x) + f(x) = 2(1-x) - (1-x)^4$$

kelib chiqadi.

$$\begin{cases} x^2 f(x) + f(1-x) = 2x - x^4 \\ (1-x)^2 f(1-x) + f(x) = 2(1-x) - (1-x)^4 \end{cases}$$

Sistemani ishlab

$$f(x) = 1 - x^2$$

Ekanligini olamiz.

4. Agar $(x-y)\,f\,(x+y) - (x+y)\,f\,(x-y) = 4xy(x^2 - y^2)\ \forall\, x, y \in \mathrm{R}$ bo'lsa, $f(x)$ funksiyani toping.

Yechim: Tenglikni ikkala tomonini $(x^2 - y^2)$ ga bo'lib yuborib

$$\frac{f(x+y)}{x+y} - \frac{f(x-y)}{x-y} = 4xy$$

Ega bo'lamiz.

$$4xy = (x+y)^2 + (x-y)^2$$

Ekanligidan foydalanib

$$\frac{f(x+y)}{x+y} - (x+y)^2 = \frac{f(x-y)}{x-y} - (x-y)^2$$

yozib olamiz. Demak

$$\frac{f(t)}{t} - t^2$$

o'zgarmas son bundan

$$\frac{f(x)}{x} - x^2 = c$$
$$f(x) = x^2 + cx$$

kelib chiqadi.

5. Agar

$$f: R - \{0,1\} \to R, f(x) + f\left(\frac{x-1}{x}\right) = 1 + x$$

bo'lsa, $f(x)$ funksiyani toping.

Yechim: x ni $\frac{x-1}{x}$ bilan almashtirib olamiz, bu yerdan

$$f\left(\frac{x-1}{x}\right) + f\left(\frac{\frac{x-1}{x}-1}{\frac{x-1}{x}}\right) = 1 + \frac{x-1}{x}$$
$$f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-x}\right) = \frac{2x-1}{x}$$

ega bo'lamiz, endi $x$ni $\frac{1}{1-x}$ bilan almashtirib

$$f\left(\frac{1}{1-x}\right) + f\left(\frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}\right) = 1 + \frac{1}{1-x} = \frac{2-x}{1-x}$$
$$f\left(\frac{1}{1-x}\right) + f(x) = \frac{2-x}{1-x}$$

ega bo'lamiz.

$$f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-x}\right) = \frac{2x-1}{x}$$
$$f\left(\frac{1}{1-x}\right) + f(x) = \frac{2-x}{1-x}$$

tengliklardan

$$f\left(\frac{x-1}{x}\right) - f(x) = \frac{2x-1}{x} - \frac{2-x}{1-x}$$

hosil bo'ladi.

$$f(x)+f\left(\frac{x-1}{x}\right)=1+x$$
$$f\left(\frac{x-1}{x}\right)-f(x)=\frac{2x-1}{x}-\frac{2-x}{1-x}$$

tengliklardan

$$f(x)=\frac{x^3-x^2-1}{2x(x-1)}$$

hosil bo'ladi.

6. Agar

$$(x-2)f(x-2)+f(2x)+f(x+2)=x+6$$

bo'lsa, $f(4)$ ni toping.

Yechim: $x=2$ almashtirish kiritamiz:

$$(2-2)f(2-2)+f(4)+f(4)=2+6$$

$2f(4)=8$ bundan $f(4)=4$ kelib chiqadi

7. Agar $3f(x)=f(x+1)+f(x-1), f(1)=3, f(2)=4$ bo'lsa, $f(5)$ ni toping.

Yechim: $x=2$ almashtirish kiritamiz $3f(2)=f(3)+f(1)$ bundan $f(3)=9$ bo'ladi.

$x=3$ almashtirish kiritamiz $3f(3)=f(4)+f(2)$ bundan $f(4)=23$ bo'ladi.

$x=4$ almashtirish kiritamiz $3f(4)=f(5)+f(3)$ bundan $f(5)=60$ bo'ladi.

8. Agar $f(x)$ funksiya uchun $x\in(-\infty;+\infty)$ da

$$f(x+3)=-\frac{1}{f(x+1)}$$

tenglik bajarilsa, $\frac{f(4)}{f(0)}$ ni toping.

Yechim: $x=1$ da

$$f(4)=-\frac{1}{f(2)}$$

bo'ladi.

$x=-1$ da

$$f(2)=-\frac{1}{f(0)}$$

bo'ladi. Bu tengliklardan

$$\frac{f(4)}{f(0)}=1$$

kelib chiqadi.

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