THE SOLUTION OF SOME GEOMETRICAL PROBLEMS OF REFERATIVE CHARACTER

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Annotation: *In this article, the solutions of some problems that belong to the analytical geometry part of geometry, are not included in the school geometry course and the higher geometry course, and are considered non-standard. That is, the solution to the problematic questions related to rectangles with a relatively high level of complexity was found analytically. The considered issues serve to develop the geometric worldview of students who want to master analytical geometry in depth.*

Keywords: *non-standart problems, analytic geometry, rectangles, bissektor, parallelogram, The quadrilateral, fashion similarly, rhombus.*

Referativ xarakterdagi ba'zi geometrik masalalarning yechimi

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Annotatsiya

Ushbu maqolada geometriyaning analitik geometriya qismiga tegishli bo_lgan, maktab geometiya kursi va oliy geometriya kursiga kiritilmagan, hamda nostandart hisoblangan ayrim masalalarning yechimlari yoritilgan. Ya‘ni, to‘rtburchaklar bilan bog_liq murakkablik darajasi nisbatan yuqori bo_lgan muommoli savollarga analitik usulda yechim topilgan. Qaralgan masalalar analitik geometriyani chuqur o_zlashtirmoqchi bo_lgan talabalarning geometrik dunyoqarashini rivojlantirishga xizmat qiladi.

Kalit so'zlar. nostandart masalalar, analitik geometriya, to'rtburchaklar, bissektrisa, parallelogramm, to'rtburchak, moda shunga o'xshash, romb.

Решение некоторых геометрических задач эталонного характера

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Аннотация

В данной статье решения некоторых задач, относящихся к аналитической геометрической части геометрии, не включены в школьный курс геометрии и высший курс геометрии и считаются нестандартными. То есть решение проблемных вопросов, связанных с прямоугольниками сравнительно высокого уровня сложности, было найдено аналитически. Рассмотренные вопросы служат развитию геометрического мировоззрения студентов, желающих углубленно освоить аналитическую геометрию.

Ключевые слова. нестандартные задачи, аналитическая геометрия, прямоугольники, биссектриса, параллелограмм, четырѐхугольник, подобие, ромб.

Introduction. In this work, the problems related to the internal properties of a square, a square and a parallelogram were solved. The goal of finding a solution to the problems presented in the article in an analytical way is set, and for this, the task of using the necessary fundamental properties is assigned. The solved problems are completely non- standard and are relevant for the development of geometric imagination for students of mathematics in higher education.

Difenation. A triangle is a simple closed curve or polygon which is created by three line-segments. In geometry, any three points, specifically non-collinear,form a unique triangle and separately, a unique plane [1:179].

The SAS Similarity Theorem. Given a correspondence be-tween two triangles. If two pairs of corresponding sides are proportional, and the included angles are congruent, then the correspondence is a similarity[1:189].

The ASA Similarity Theorem. If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent[5:220].

***Theorem-1*.**

The trisectors of the angles of a rectangle are drawn. For each pair of adjacent angles, those triseciors that are closest to the enclosed side are extended until a point of intersection is established. The line segments connecting those points of intersection form a quadrilateral. Prove that the quadrilateral is a rhombus.

Proof. As a result of the trisections,

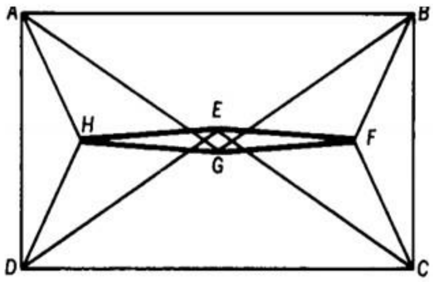
isosceles $\Delta AHD \approx$ isosceles ΔBFC, and isosceles $\Delta AGB \approx$ isosceles ΔDEC (Fig. S-2).

Since $AH = HD = FB = FC$, and $AG = GB = DE = CE$,

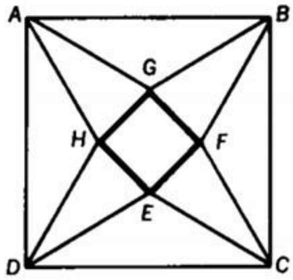
and $\angle HAG \cong \angle GBF \cong \angle FCE \cong \angle HDE \cong \frac{1}{3}$ right angle,

$\Delta HAG \cong \Delta FBG \cong \Delta FCE \cong \Delta HDE$ (S.A.S.).

Therefore, $HG = FG = FE = HE$, and $EFGH$ is a rhombus.



(S-2)



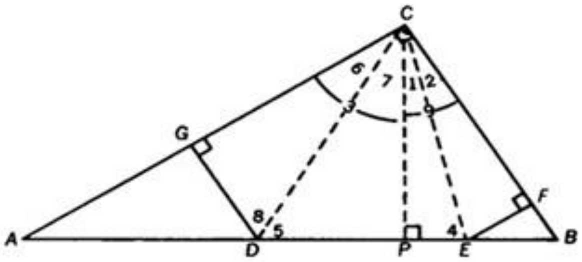
(S-3)

Challenge 1 *What type of quadrilateral would be formed if the original rectangle were replaced by a square?*

Consider ABCD to be a square (Fig. S-3). All of the above still holds true; thus we still maintain a rhombus. However, we now can easily show $\Delta\ AHG$ to be isosceles, $m\angle AGH = m\angle AHG = 75°$, Similarly, $m\angle BGF = 75°$. $m\angle AGB = 120°$, since $m\angle GAB = m\angle GBA = 30°$. Therefore, $m\angle HGF = 90°$. We now have a rhombus with one right angle; hence, a square.

Theorem-2.

In right $\Delta\ ABC$, *with right angle at* $C, BD = BC, AE = AC$, $\overline{EF} \perp \overline{BC}$, *and* $\overline{DG} \perp \overline{AC}$. *Prove that* $DE = EF + DG$.



(S-4)

Proof. Draw $\overline{CP} \perp \overline{AB}$, also draw $\overline{CE}$ and $\overline{CD}$ (Fig. S-4).

$$m\angle 3 + m\angle 1 + m\angle 2 = 90^\circ$$
$$m\angle 3 + m\angle 1 = m\angle 4 \ (\#5)$$

By substitution,

$$m\angle 4 + m\angle 2 = 90^\circ$$

but in right $\Delta\ CPE$, $m\angle 4 + m\angle 1 = 90^\circ$.

Thus, $\angle 1 \cong \angle 2$ (both are complementary to $\angle 4$), and right $\Delta\ CPE \cong right\ \Delta\ CFE$, and $PE = EF$. Similarly,

$$m\angle 9 + m\angle 7 + m\angle 6 = 90^\circ$$
$$m\angle 9 + m\angle 7 = m\angle 5 .$$

By substitution, $m\angle 5 + m\angle 6 = 90^\circ$. However, in right $\Delta\ CPD$,

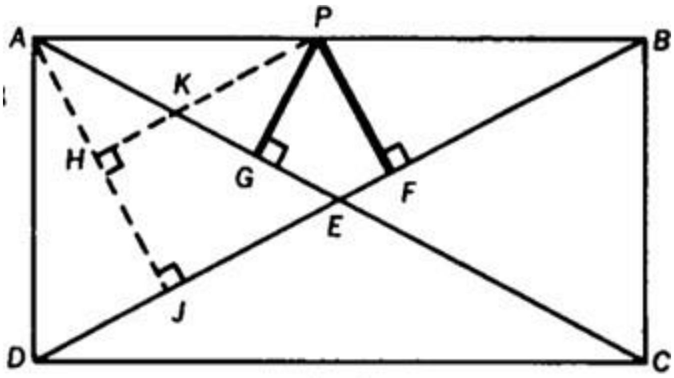
$$m\angle 5 + m\angle 7 = 90^\circ$$

Thus, $\angle 6 \cong \angle 7$ (both are complementary to $\angle 5$), and right

$\Delta CPD \cong right\ \Delta CGD$ and $DP = DG$.
Since $DE = DP + PE$, we get $DE = DG + EF$.

Theorem-3.

Prove that the sum of the measures of the perpendiculars from any point on a side of a rectangle to the diagonals is constant.



(S-5)

Proof. Let P be any point on side $\overline{AB}$ of rectangle ABCD (Fig. S-5).

$\overline{PG}$ and $\overline{PF}$ are perpendiculars to the diagonals. Draw $\overline{AJ}$ perpendicular to $\overline{BD}$ and then $\overline{PH}$ perpendicular to $\overline{AJ}$. Since $PHJF$ is a rectangle (a quadrilateral with three right angles),we get $PF = HJ$. Since $\overline{PH}$ and $\overline{BD}$ are both perpendicular to $\overline{AJ}$, $\overline{PH}$ is parallel to $\overline{BD}$. Thus, $\angle APH \cong \angle ABD$. Since, $AE = EB$, $\angle CAB \cong \angle ABD$. Thus, by transitivity, $\angle EAP \cong \angle APH$; also in ΔAPK, $AK = PK$. Since $\angle AKH \cong \angle PKG$, right $\Delta AHK \cong right\ \Delta PGK$ (S.A.A.). Hence, $AH = PG$ and, by addition, $PF + PG = HJ + AH = AJ$, a constant.

Theorem 4.

Given square $ABCD$ with $m\angle EDC = m\angle ECD = 15°$, prove ΔABE is equilateral.

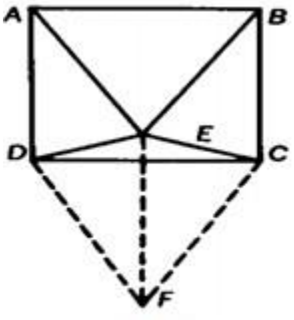
Method 1: In square $ABCD$, with $m\angle EDC = m\angle ECD = 15°$ draw ΔAFD on $\overline{AD}$ such that $m\angle FAD = m\angle FDA = 15°$. Then draw $\overline{FE}$ (S-7).

$\Delta FAD \cong \Delta EDC$, and $DE = DF$.

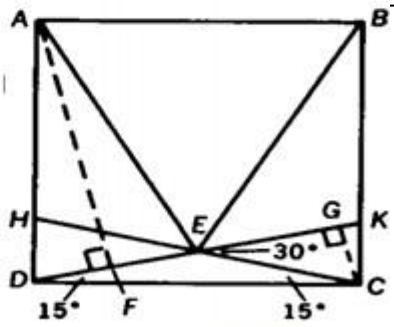
Since $\angle ADC$ is a right angle, $m\angle FDE = 60°$ and ΔFDE is equilateral so that $DF = DE = FE$. Since $m\angle DFE = 60°$ and $m\angle AFD = 150°$, $m\angle AFE = 150°$. Thus, $m\angle FAE = 15°$ and $m\angle DAE = 30°$. Therefore, $m\angle EAB = 60°$. In a similar fashion it may be proved that $m\angle ABE = 60°$; thus, ΔABE is equilateral.

Method 2: In square $ABCD$, with $m\angle EDC = m\angle ECD = 15°$, draw equilateral ΔDFC on $\overline{DC}$ externally; then draw $\overline{EF}$ (S - 7). $\overline{EF}$ is the perpendicular bisector of $\overline{DC}$

Since $AD = FD$, and $m\angle ADE = m\angle FDE = 75°$, $\Delta ADE \cong \Delta FDE$. Since $m\angle DFE = 30°$, $m\angle DAE = 30°$. Therefore, $m\angle BAE = 60°$. In a similar fashion, it may be proved that $m\angle ABE = 60°$; thus, ΔABE is equilateral.



(S-8)



(S-9)

E

Method 3: Extend $\overline{DE}$ and $\overline{CE}$ to meet $\overline{BC}$ and $\overline{AD}$ at K and H, respectively (Fig.S-8) In square $ABCD$, $m\angle KDC = m\angle HCD = 15°$, therefore, $ED = EC$. Draw $\overline{AF}$ and $\overline{CG}$ perpendicular to $\overline{DK}$ In right $\Delta\, DGC$, $m\angle GCD = 75°$, while $m\angle ADF = 75°$ also. Thus, $\Delta\, ADF \cong \Delta\, DCG$, and $DF = CG$. $m\angle GEC = 30°$. In $\Delta\, GEC$, $CG = \frac{1}{2}(EC)$. Therefore, $CG = \frac{1}{2}(ED)$, or $DF = \frac{1}{2}(ED)$. Since $\overline{AF}$ is the perpendicular bisector of DE, $AD = AE$. In a similar fashion, it may be proved that $BE = BC$; therefore, $\Delta\, ABE$ is equilateral.

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