OLIMPIADAGA TAYYORGARLIK JARAYONIDA FOYDALANILADIGAN YIG'INDILAR

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Annotatsiya: *Ushbu maqolada olimpiadaga tayyorgarlik ko'rayotgan o'quvchilarning mashaqqatli mehnatini salgina bo'lsa ham osonlashtirishga urg'u berilgan. Maqolada asosan yig'indilarni qiymatini topishga doir misollar ko'rib chiqilgan. Matematika fanida yig'indilarni hisoblashni bir qancha usullari mavjud bo'lib quyida shularga to'xtalib o'tamiz.*

Kalit so'zlar: *Yig'indi, faktorial, kasr.*

Misollarga o'tishdan oldin Matematika fanining rivojlanish jarayonidagi "Musulmon renessansi" deb nomlangan yuksalish davriga to'xtalib o'tamiz.

IX asrdan fan tarixi "Musulmon renessansi" deb nomlangan yangi yuksalish davriga kiradi. "Bayt ul–hikmat" da Yunoniston, Hindiston, Xorazm va Xitoyda jamg'arilgan bilimlar sintez qilinib, matematika izchil rivojlantirila boshlandi. Muso al–Xorazmiy tomonidan 0 raqamining ixtiro qilinishi va bu orqali o'nli sanoq sistemasining tartibga keltirilishi butun jahon matematikasining olamshumul yutug'i bo'ldi va buning natijasida u algebraga asos soladi. Uning o'nli sanoq sistemasi bayon qilingan asari tufayli bu qulay hisoblash vositasi dunyoga yoyildi. Asarlari o'qimishli bo'lishi uchun Xorazmiy aniq va lo'nda bayon uslubini qo'llagan. Shu tufayli uning asarlari keng tarqalgan. Xorazmiy uslubi yevropalik tarjimonlar tomonidan muallif nomi bilan algoritm deb atalgan.

Musulmon Sharqi olimlari geometriyani ham rivojlantirgan (Sobit ibn Qurra, Abulvafo, Umar Xayyom), trigonometriyaga fan sifatida asos solganlar (Ibn al-Xaysam, Beruniy, Tusiy), xususan, Ahmad al–Farg'oniy tomonidan Ptolomeyning stereografik proyeksiya haqidagi teoremasining isbotlanishi Bag'dod akademiyasida geometriya chuqur o'rganilganini ko'rsatdi. Arab tilida ijod qilgan matematiklarning uchinchi va to'rtinchi darajali tenglamalarni geometrik usulda yechish yo'llari keyinchalik analitik geometriya yaratilishiga turtki bo'lgan. Matematika rivojlanishida Xorazm Ma'mun akademiyasi (Ibn Iroq, Beruniy) ham muhim rol o'ynagan. Sharq matematikasi rivojining cho'qqisi esa Samarqand ilmiy maktabi davriga to'g'ri keladi. Ulug'bek va uning rahbarligidagi olimlar (Qozizoda Rumiy, G'iyosiddin Koshiy, Ali Qushchi, Miram Chalabiy, Husayn Birjaniy va b.) ulkan rasadxona qurish, yulduzlar koordinatalari va sayyoralar harakatini katta aniqlikda kuzatish ishlari bilan birga kuzatuv natijalari bo'yicha yoritqichlarning sferik koordinatalarini hisoblash usullarini, interpolyasiya formulalari, keyinchalik Gorner sxemasi deb atalgan usulni hamda ketma–ket yaqinlashishlar usulini ishlab chiqadilar. Ulug‘bekning “Ziji jadidi Ko‘ragoniy” asaridan o‘ta aniqlikdagi trigonometrik funksiyalar jadvallari ham o‘rin olgan.

1-MISOL. Yig’indining qiymatini toping.

$$S=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{100\cdot 101}$$

YECHIM: Bu ko’rinishdagi misollar asosan quyidagi usulda yechiladi. Har bir kasrni quyidagicha yozib olamiz

$$\frac{1}{1\cdot 2}=\frac{1}{1}-\frac{1}{2},\frac{1}{2\cdot 3}=\frac{1}{2}-\frac{1}{3},\ldots,\quad \frac{1}{100\cdot 101}=\frac{1}{100}-\frac{1}{101}$$

Ya’ni, umumiy holda

$$\frac{1}{k\cdot(k+1)}=\frac{1}{k}-\frac{1}{k+1}$$

Bundan ko’rinib turibdiki yig’indining javobi

$$S=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{100}-\frac{1}{101}=1-\frac{1}{101}=\frac{100}{101}$$

Ekan.

2-MISOL. Yig’indini hisoblang.

$$S=\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\frac{1}{9\cdot 13}+\cdots+\frac{1}{101\cdot 105}=?$$

YECHIM: Yuqoridagi misoldagi kabi har bir kasrni quyidagicha yozib olamiz

$$\frac{1}{1\cdot 5}=\frac{1}{4}\left(\frac{1}{1}-\frac{1}{5}\right),\frac{1}{5\cdot 9}=\frac{1}{4}\left(\frac{1}{5}-\frac{1}{9}\right),\ldots,\quad \frac{1}{101\cdot 105}=\frac{1}{4}\left(\frac{1}{101}-\frac{1}{105}\right)$$

Ya’ni, umumiy holda

$$\frac{1}{(4k+1)\cdot(4k+5)}=\frac{1}{4}\left(\frac{1}{4k+1}-\frac{1}{4k+5}\right)$$

Umumiy holdan foydalansak

$$S=\frac{1}{4}\left(1-\frac{1}{5}+\frac{1}{5}-\frac{1}{9}+\frac{1}{9}-\frac{1}{13}+\cdots+\frac{1}{101}-\frac{1}{105}\right)=\frac{1}{4}\left(1-\frac{1}{105}\right)=\frac{26}{105}$$

Ekan.

Mustaqil ishlash uchun: Yig’indini hisoblang.

$$S=\frac{1}{11\cdot 4}+\frac{1}{18\cdot 11}+\frac{1}{25\cdot 18}+\cdots+\frac{1}{102\cdot 95}=?$$

Javob: $\frac{7}{204}$

3-MISOL. Yig’indini hisoblang

$$S=\frac{1}{1\cdot 4\cdot 7}+\frac{1}{4\cdot 7\cdot 10}+\frac{1}{7\cdot 10\cdot 13}+\cdots+\frac{1}{25\cdot 28\cdot 31}$$

YECHIM: Har bir kasrni quyidagicha yozib olamiz

$$\frac{1}{1\cdot 4\cdot 7}=\frac{1}{6}\left(\frac{1}{1\cdot 4}-\frac{1}{4\cdot 7}\right)$$

$$\frac{1}{4\cdot 7\cdot 10}=\frac{1}{6}\left(\frac{1}{4\cdot 7}-\frac{1}{7\cdot 10}\right)$$

$$\frac{1}{7\cdot 10\cdot 13}=\frac{1}{6}\left(\frac{1}{7\cdot 10}-\frac{1}{10\cdot 13}\right)$$

$$\vdots$$

$$\frac{1}{25\cdot 28\cdot 31}=\frac{1}{6}\left(\frac{1}{25\cdot 28}-\frac{1}{28\cdot 31}\right)$$

Ya’ni, yuqoridagilardan foydalanib

$$S=\frac{1}{6}\left(\frac{1}{1\cdot 4}-\frac{1}{28\cdot 31}\right)=\frac{9}{217}$$

Ekanligini topamiz.

Mustaqil ishlash uchun: Yig’indini hisoblang.

$$S=\frac{8}{41\cdot 49\cdot 45}+\frac{8}{37\cdot 45\cdot 41}+\frac{8}{33\cdot 41\cdot 37}+\cdots+\frac{8}{1\cdot 9\cdot 5}=?$$

Javob: $\frac{88}{441}$

4-MISOL. Yig’indini hisoblang.

$$S=1+2\cdot 3+3\cdot 7+\cdots+n(2^n-1)=?$$

YECHIM: Har bir qo’shiluvchini quyidagicha yozib olamiz

$$S=(1\cdot 2-1)+(2\cdot 2^2-2)+(3\cdot 2^3-3)+\cdots+(n\cdot 2^n-n)$$

Guruhga ajratamiz

$$S=(1\cdot 2+2\cdot 2^2+3\cdot 2^3+\cdots+n\cdot 2^n)-(1+2+3+\cdots+n)$$

$$S=S_n-S_k$$

$$S_n=1\cdot 2+2\cdot 2^2+3\cdot 2^3+\cdots+n\cdot 2^n$$

$$\frac{S_n}{2}=1+2\cdot 2+3\cdot 2^2+\cdots+n\cdot 2^{n-1}$$

$$\frac{S_n}{2}-S=1+2+2^2+2^3+\cdots+2^{n-1}-n\cdot 2^n$$

$$S_k=1+2+3+\cdots+n=\frac{1+n}{2}\cdot n$$

$$S_n-S_k=2^{n+1}(n-1)+2-\frac{(n+1)\cdot n}{2}$$

Yuqorida bajarilgan amallardan foydalanib, quyidagiga ega bo’lamiz.

$$S_n=2^{n+1}\cdot(n-1)+2$$

5-MISOL Yig’indini hisoblang.

$$S=\frac{1}{1\cdot 3}+\frac{2}{1\cdot 3\cdot 5}+\frac{3}{1\cdot 3\cdot 5\cdot 7}+\frac{4}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\cdots=?$$

YECHIM:

$$S=\frac{1}{1\cdot 3}+\frac{2}{1\cdot 3\cdot 5}+\frac{3}{1\cdot 3\cdot 5\cdot 7}+\frac{4}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\cdots=\sum_{n=1}^{\infty}\frac{n}{(2n+1)!!}$$

$$\frac{1}{(2n-1)!!}-\frac{1}{(2n+1)!!}=\frac{2n}{(2n+1)!!}$$

$$\frac{n}{(2n+1)!!}=\frac{1}{2}\left(\frac{1}{(2n-1)!!}-\frac{1}{(2n+1)!!}\right)$$

$$\sum_{n=1}^{\infty}\frac{n}{(2n+1)!!}=\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{(2n-1)!!}-\frac{1}{(2n+1)!!}\right)=\frac{1}{2}\left(\frac{1}{1!!}-\frac{1}{3!!}+\frac{1}{3!!}+\cdots-\frac{1}{(2n+1)!!}\right)$$

DEMAK

$$\sum_{N=1}^{\infty}\frac{n}{(2n+1)!!}=\frac{1}{2}\left(1-\frac{1}{(2n+1)!!}\right)$$

Ekan.

6-MISOL. Yig’indini hisoblang.

$$S=1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots+100\cdot 100!$$

YECHIM:

$$k\cdot k!=\big((k+1)-1\big)k!=(k+1)!-k!$$

Dan foydalanamiz

$$S=(2!-1!)+(3!-2!)+(4!-3!)+\cdots+(101!-100!)$$

Demak

$$S=101!-1$$

Ekan

7-MISOL

$$S=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}=?$$

YECHIM:

$$\frac{n}{(n+1)!}=\frac{(n+1)-1}{(n+1)!}=\frac{1}{n!}-\frac{1}{(n+1)!}$$

Ekanligidan foydalanamiz

$$S=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}$$

$$=\left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\left(\frac{1}{3!}-\frac{1}{4!}\right)+\cdots+\left(\frac{1}{n!}-\frac{1}{(n+1)!}\right)=1-\frac{1}{(n+1)!}$$

MUSTAQIL ISHLASH UCHUN

$$S=\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\frac{5}{3!+4!+5!}+\cdots+\frac{100}{98!+99!+100!}=?$$

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