

TRIGONOMETRIK FUNKSIYALAR MODULINI O'QITISH METODIKASINI INTERFAOL METODLAR ASOSIDA TAKOMILLASHTIRISH

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Annotatsiya: *Ushbu maqolada trigonometrik funksiyalarni o'rganish usullari misollari haqida so'z yuritiladi.*

Kalit so'zlar: *funksiyalar, $\sin x$, $\cos x$, ifoda integratsiyasi, ratsional.*

Trigonometrik funksiyalar qatnashgan ifodalarni integrallash

Hamma trigonometrik funksiyalarni $\sin x$ va $\cos x$ orqali ratsional ko'rinishda ifodalash mumkin. Bu ifodani $R(\sin x, \cos x)$ orqali belgilaymiz.

Endi $R(\sin x, \cos x)$ ko'rinishdagi ifodani integrallash kerak bo'lsin.

Bunday integralni z belgilash yordamida z o'zgaruvchili ratsional funksiyaning integraliga almashtirish mumkin. Integralni bunday almashtirish ratsionallashtirish deyiladi. Haqiqatdan ham, $\int \frac{dx}{\sin x \cos x} = \int \frac{dx}{\sin x} - \int \frac{dx}{\cos x} = -\ln|\cos x| - \ln|\sin x| + C = -\ln|\sin x \cos x| + C$. Shuning uchun $\int R(\sin x, \cos x) dx = \int R(z) dz$ bunda $R(z)$ - z o'zgaruvchili ratsional funksiya.

Bunday almashtirish $R(\sin x, \cos x)$ ko'rinishdagi har qanday funksiyani integrallashga imkon beradi, shuning uchun bunday almashtirish universal trigonometrik almashtirish deyiladi. Lekin bunday almashtirish ko'pincha ancha murakkab ratsional funksiyaga olib keladi.

Shuning uchun, sodda o'rniga qo'yishlardan ham foydalansa bo'ladi.

Masalan:

1) Agar $R(\sin x, \cos x)$ funksiya $\sin x$ ga nisbatan toq bo'lsa, ya'ni $R(\sin x, -\cos x) = R(\sin x, \cos x)$ bo'lsa, u holda $z = \sin x$; $dz = \cos x dx$ o'rniga o'miga qo'yish Bu holda IV.UZ ratsionallashtiradi. Bu holda bu funksiyani bo'ladi.

$R(-\sin x, \cos x) = R(\sin x, \cos x)$ bo'lsa, u holda $z = \cos x$; $dz = -\sin x dx$ o'miga qo'yish bu funksiyani ratsionallashtiradi. 2) Agar $R(\sin x, \cos x)$ funksiya $\cos x$ ga nisbatan toq bo'lsa, ya'ni qo'yish bu funksiyani ratsionallashtiradi. 3) Agar $R(\sin x, \cos x)$ funksiya $\sin x$ va $\cos x$ ga nisbatan juft bo'lsa, ya'ni $R(-\sin x, -\cos x) = R(\sin x, \cos x)$ bo'lsa, Holda

$z = \tan x$; $dz = \frac{dx}{\cos^2 x}$ Fixareigz

$\int \frac{1 + \tan^2 x}{\cos x} dx$

dx

$4 \sin x + 3 \cos x + 5$

1-Misol integralni hisoblang.

Yechish:

=o'rniga qo'yishdan foydalanamiz.

1- 22

2dz

$$2dz \quad 22 \quad 1+22 \quad 22 \quad +82 \quad +8 \quad +3 \quad 1-22 \quad 1+22 \quad f \quad 2dz \quad 8z+3-322+5+5^2 \quad 1+22 \quad +5 \quad dz$$

2-Misol. $J = dx$ integralni hisoblang.

Yechish: Integral belgisi ostidagi funksiya juft funksiya, shuning uchun $tgx=z$ almashtirishni bajaramiz.

U holda $z=tgx$; $x=arctgz$; $dx = \frac{dz}{1+z^2}$

Natijada quyidagini hosil qilamiz:

$$dz \quad J = \int \frac{1}{1+z^2} dz = \arctg z + C = \arctg \sqrt{2x} + C$$

3-Misol: $J = \sin x dx$ integralni hisoblang.

Yechish: Integral ostidagi funksiya $\sin x$ ga nisbatan toq funksiya. Shuning uchun $z=\cos x$; $dz=-\sin x dx$ $\sin x dx = -dz$ almashtirishni bajaramiz:

$$J = \int \sin x dx = -\cos x + C$$

4) Agar $R(\sin x, \cos x)$ funksiya $\sin x$ va $\cos x$ darajalarining ko'paytmasi bo'lsa, ya'ni $\sin^m x \cos^n x$ ko'rinishdagi integralni hisoblash, m va n ga bog'liq holda turli o'rniga qo'yishlar bajariladi: a) Agar $n > 0$ va toq bo'lsa, u holda $z = \cos x$ bo'lsa, u holda $\cos^n x - z^n$; $\sin x dx = -dz$ o'miga qo'yish integralni ratsionallashtiradi. b) Agar $m > 0$ va toq bo'lsa, u holda $\sin x - z$; $\cos x dx = dz$ o'rniga qo'yish bajariladi.

4-Misol: $\int \cos^4 x dx$ integralni hisoblang.

Yechish: $\cos^4 x - z^4$; $\sin x dx = -dz$ almashtirishni bajaramiz:

$$J = \int \cos^4 x dx = \int (1 - \sin^2 x)^2 dx = \int (1 - 2\sin^2 x + \sin^4 x) dx = \int (1 - 2(1 - \cos^2 x) + \cos^4 x) dx = \int (3\cos^2 x - 2 + \cos^4 x) dx = \int 3\cos^2 x dx - \int 2 dx + \int \cos^4 x dx = \frac{3}{2} \sin 2x - 2x + \int \cos^4 x dx + C$$

v) Agar ikkala n va m ko'rsatkichlar juft va nomanfiy bo'lsa, u holda trigonometriyadan ma'lum bo'lgan

$$\sin^2 x = \frac{1 - \cos 2x}{2}; \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

darajani pasaytirish formulalaridan foydalanamiz. 5-Misol. - $\int \sin^4 x dx$ integralni hisoblang.

Yechish: Darajani pasaytirish formulasidan foydalanamiz.

$$J = \int \sin^4 x dx = \int (1 - \cos^2 x)^2 dx = \int (1 - 2\cos^2 x + \cos^4 x) dx = \int (1 - 2 \cdot \frac{1 + \cos 2x}{2} + \frac{(1 + \cos 2x)^2}{4}) dx = \int (1 - 1 - \cos 2x + \frac{1 + 2\cos 2x + \cos^2 2x}{4}) dx = \int (\frac{1 - 2\cos 2x + 1 + 2\cos 2x + \cos^2 2x}{4}) dx = \int (\frac{2 + \cos^2 2x}{4}) dx = \frac{1}{2} \int (1 + \frac{1 + \cos 4x}{2}) dx = \frac{1}{2} \int (1 + \frac{1}{2} + \frac{\cos 4x}{2}) dx = \frac{1}{2} \int (\frac{3}{2} + \frac{\cos 4x}{2}) dx = \frac{3}{4} x + \frac{1}{8} \sin 4x + C$$

6-Misol. $\int \frac{dx}{\sin^3 x \cos x}$ integralni hisoblang

Yechish: bu yerda $n=-3$; $m=-1$; $m+n=-4 < 0$

$$J = \int \frac{dx}{\sin^3 x \cos x} = \int \frac{(\sin^2 x + \cos^2 x) dx}{\sin^3 x \cos x} = \int \frac{\sin^2 x}{\sin^3 x \cos x} dx + \int \frac{\cos^2 x}{\sin^3 x \cos x} dx = \int \frac{1}{\sin x \cos x} dx + \int \frac{\cos x}{\sin^3 x} dx = 2 \int \sin 2x dx + \int d(\sin x) \sin^{-2} x = -\cos 2x + \frac{1}{\sin x} + C$$

7-Misol.

An integralni hisoblang. Yechish: bu yerda $n=2$, $m=-6$; $n+m=-4 < 0$ quyidagini almashtirishni

$z = \operatorname{tg} x; x = \operatorname{arctg} z; dx = dz \cdot \frac{1}{1+z^2}$

Natijada quyidagini hosil qilamiz.

$\int \frac{dx}{1+z^2} = \operatorname{arctg} z + C$

8-Misol. $\int \frac{dx}{1+z^2}$ integralni hisoblang.

Yechish: bu yerda $m=4; n=-4; m+n=0$;

Quyidagi almashtirishni bajaramiz.

$z = \operatorname{ctg} x; x = \operatorname{arctg} z; dz = -dx$

Natijada

$\int \frac{dx}{1+z^2} = -\operatorname{arctg} z + C$

9-Misol. $\int \frac{dx}{1+z^2}$ integralni hisoblang. Yechish bu yerda $n=0; m=-6; m+n=-6 < 0$ quyidagi almashtirishni bajaramiz.

U holda $dx = \frac{dz}{1+z^2}$ Agar darajalardan biri nolga teng, ikkinchisi manfiy toq son bo'lsa, u holda almashtirish bajariladi.

10-Misol. $\int \frac{dx}{1+z^2}$ integralni hisoblang.

Yechish: Quyidagicha almashtirishni bajaramiz.

Natijada:

$\int \frac{dx}{1+z^2} = \operatorname{arctg} z + C$

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