PARABOLIK TIPDAGI CHIZIQSIZ TENGLAMA UCHUN YECHIMNING ASIMPTOTIKASI

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Ushbu bobda, chiziqsiz ajratish algoritmi [1] yordamida, $Q = \{(x,t): x \in R^N, t > 0\}$ sohada birinchi chegaraviy masala qaralgan.

$$Lu \equiv -\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\left|\frac{\partial u^k}{\partial x}\right|^{n-1}\frac{\partial u^k}{\partial x}\right) - \gamma(t)u^\beta, \tag{2.10}$$

$$u\big|_{t=0} = u_0(x) \geq 0,\ x \in R_+^1,\quad u\big|_{x=0} = \psi(t),\quad t>0,\quad \psi(t) \in C^1(R_+^1), \tag{2.11}$$

$$\beta, kn > 1,\quad x \in R_+^1,\quad t \in R_+^1,\quad 0 < \gamma(t) \in C(R_+^1).$$

(2.10), (2.11) masala koeffitsientlarning berilgan miqdori va uning $\frac{\partial u}{\partial x}$ gradiyenti quvvatiga bog'liq bo`lgan, chiziqli bo'lmagan issiqlik tarqalish jarayonining matematik modeli hisoblanadi [2]. (2.10) tenglama boshqa jarayonlarni ham tasvirlaydi [3]. Ayniqsa, n = 1 uchun (2.10), (2.11) masala chiziqsiz issiqlik tarqalish jarayonini, va $k, n \neq 1$ qiymatlar uchun turbulent filtratsiya jarayonlarini tasvirlaydi [4]. (2.10), (2.11) masalani o'rganishning turli jihatlari ko'plab mualliflar tomonidan ko'rib chiqilgan va kutilmagan chiziqsiz ta'sirlar olingan (qarang [3] va u yerda keltirilgan adabiyot).

(2.10) tenglama buzuladigan bo`lganligi uchun, $u\,(x,t) = 0$ yechim klassik ma'noda sohada joylashmagan bo`lishi mumkin. Shuning uchun (2.10) ning umumlashtirilgan echimlarni ko'rib chiqish kerak, ammo amaliyot uchun fizik ma'noga ega bo'lgan umumiy yechimlarni o'rganish maqsadga muvofiqdir.

$$0 \leq u(t,x) \in C(Q),\quad \left|\frac{\partial u^k}{\partial x}\right|^{n-1} \cdot \frac{\partial u^k}{\partial x} \in C(Q).$$

(2.10), (2.11) masalaning umumiy yechimi taqsimot ma'nosida tushuniladi [3].

Shuni nazarda tutamizki, k = 1 yoki n = 1 hollaridagi (2.10) tenglama uchun Koshi masalasi ko'plab mualliflar tomonidan ilgari o'rganilgan [5, 6]. k> 1, n> 1 holi ishi [2] da ko'rib chiqildi. Bizning yondashuvimiz mualliflar tomonidan rasmiy ravishda tuzilgan yechimdan farq qiladi [7, 8] va (2.10), (2.11) birinchi chegaraviy masala uchun chiziqsiz ajratish algoritmini qo'llashga asoslangan. $\beta = 1,\ k = 1,\ n > 1,$ bo'lsa, lahzali manba muammosining avtomodel yechimlari $\left(u_0(x) = E\delta(x)\right)$ [9] da qaralgan, bu erda E – manba kuchi, $\delta(x)$ – Dirak delta funktsiyasi va yechim lokalizatsiya holati shaklda olingan.

$$\int_0^t exp(-(n-1)\int_0^\eta \gamma(\omega)d\omega)d\eta < +\infty.$$

Keling, nazariya va ilovalar uchun juda muhim ahamiyatga ega bo'lgan $\beta > 1$holatda yechim va old (erkin chegara) $\left(u_0(x) = E\delta(x)\right)$ (2.10), (2.11) masalasini ko'rib chiqaylik. Asosiy savol - assimilyatsiya jarayonining diffuziya jarayoniga ta'siri (2.10), (2.11) da tasvirlangan.

Boshlang'ich tenglamaning bo'linishiga asoslangan chiziqsiz ajratish algoritmi bilan, birinchi bosqichda tenglama yechiladi

$$\frac{d\bar{u}}{dt} = -\gamma(t)\bar{u}^\beta,$$

$$\bar{u}(t) = \left[1 + (\beta - 1)\int_0^t \gamma(t)dt\right]^{-\frac{1}{\beta-1}}.$$

Bu funksiya (2.10) tenglama oqimining "hissasi".

Keling, quyidagi belgilashlarni ko`rib chiqaylik

$$u_+(t,x) = \tilde{u}(t)\overline{f}(\xi), \qquad (2.12)$$

bu yerda $\bar{u}(t) = \left[1 + (\beta - 1)\int_0^t \gamma(t)dt\right]^{-\frac{1}{\beta-1}}$, $\tilde{u}(t) = \bar{u}(t)\cdot\psi(t)$,

$$\tau(t) = \begin{cases} \int_0^t [\bar{u}(t)]^{kn-1}dt, \ agar\, kn \neq 1, \\ T + t \quad , \ agar\, kn = 1, \end{cases} \qquad (2.13)$$

$$\overline{f}(\xi) = \left(a - b\xi^{\frac{n+1}{n}}\right)^{\frac{n}{kn-1}}, \qquad (2.14)$$

bu yerda $\xi = \frac{|x|}{[\tau(t)]^{\frac{1}{n+1}}}$, $b = \frac{kn-1}{k(n+1)}\left(\frac{1}{n+1}\right)^{\frac{1}{n}}$, $a > 0$, $\psi(0) > 0$.

Ikkinchi bosqichda, chiziqsiz ajratish algoritmiga ko'ra [1], (2.10), (2.11) masalaning umumlashgan yechimi quyidagi ko`rinishda qarab chiqiladi

$$u(t,x) = \tilde{u}(t)w(\tau(t),x). \qquad (2.15)$$

(2.15) ni (2.10) ga quyamiz va $\tau(t)$ ni (2.13) tanlab, $kn \neq 1$ uchun quyidagi tenglamaga ega bo`lamiz

$$\frac{\partial w}{\partial \tau}=\frac{\partial}{\partial x}\left(\left|\frac{\partial w^k}{\partial x}\right|^{n-1}\frac{\partial w^k}{\partial x}\right)-\tilde{u}_t\cdot\tilde{u}^{-kn}\cdot w-\tilde{u}^{\beta-kn}\cdot\gamma(t)\cdot w^{\beta}\ . \qquad (2.16)$$

(2.10) tenglamaning manba mavjud bo`lmagan, lekin boshqa vaqt o`zgaruvchisi bo`lgan holatini qarab chiqamiz

$$\frac{\partial w}{\partial \tau}=\frac{\partial}{\partial x}\left(\left|\frac{\partial w^k}{\partial x}\right|^{n-1}\frac{\partial w^k}{\partial x}\right), \qquad (2.17)$$

bunda $\tau(t)$ - funksiya (2.13) formuladan olingan.

Ushbu tenglama, (2.10) etalon tenglamasi deyiladi, oltita avtomodel yechimlarga ega, ulardan biri quyidagi ko`rinishga ega

$$w(\tau,x)=f(\xi),\ \text{где}\quad \xi=\frac{|x|}{[\tau(t)]^{\frac{1}{n+1}}}\ . \qquad (2.18)$$

(2.18) ni (2.16) ga quyib, quyidagi taqribiy-avtomodel tenglamaga ega bo`lamiz

$$\frac{d}{d\xi}\left(\left|\frac{df^k}{d\xi}\right|^{n-1}\frac{df^k}{d\xi}\right)+\frac{\xi}{n+1}\frac{df}{d\xi}-\tilde{u}_t\cdot\tilde{u}^{-kn}\cdot\tau\cdot f-\tilde{u}^{\beta-kn}\cdot\tau\cdot\gamma(t)\cdot f^{\beta}=0\ . \qquad (2.19)$$

(2.19) тенглама (2.10), (2.11) масаланинг глобал ечимга эгалигини кўрсатишга имкон беради.

Қуйидаги теорема ўринли:

Теорема 2.2.1. Agar *u(t,x)* (2.10) tenglamaning umumiy yechimi bo`lsa, u holda (2.10), (2.11) masalaning global yechimi mavjud va bu yechim uchun $Q=\{(t,x): t>0, x\in R_+^1\}$ sohada

$$-\tau(t)\tilde{u}_t\tilde{u}^{-kn}\leq\frac{1}{n+1};\quad u_0(x)\leq u_+(0,x)$$

shartlar bajarilganda,

$$u(t,x)\leq u_+(t,x)=\tilde{u}(t)\bar{f}(\xi)$$

baho o'rinli bo'ladi.

Isboti. (2.10)-(2.11) masalaning yuqori yechimini, quyidagi ko`rinishda izlaymiz

$$u_+(t,x)=\tilde{u}(t)\bar{f}(\xi),$$

du yerda

$$\tilde{u}(t)=\bar{u}(t)\cdot\psi(t),\qquad \bar{u}(t)=\left[1+(\beta-1)\int_0^t\gamma(t)dt\right]^{-\frac{1}{\beta-1}},$$

$\tau(t)$ va $\bar{f}(\xi)$ funksiyalar (2.13) va (2.14) formulalardan topiladi.

To`g`ridan-to`g`ri ko`zatuvlar shuni ko`rsatadiki, quyidagi funksiya

$$\bar{f}(\xi)=\left(1-\frac{kn-1}{k(n+1)}\cdot\left(\frac{1}{n+1}\right)^{\frac{1}{n}}\xi^{\frac{n+1}{n}}\right)_{+}^{\frac{n}{kn-1}}$$

bu tenglamaning umumiy yechimidir

$$\frac{d}{d\xi}\left(\left|\frac{d\bar{f}^{\,k}}{d\xi}\right|^{n-1}\frac{d\bar{f}^{\,k}}{d\xi}\right)+\frac{\xi}{n+1}\frac{d\bar{f}}{d\xi}+\frac{1}{n+1}\bar{f}=0, \qquad (2.20)$$

$\bar{f}$ yechim uzluksiz va $\left|\frac{d\bar{f}^{\,k}}{dx}\right|^{n-1}\cdot\frac{d\bar{f}^{\,k}}{dx}$ oqimining xususiyatiga ega..

Isbotlash uchun biz solishtirish teoremalaridan foydalanamiz

Quyidani isbotlaymiz

$$Lu_{+}\le 0 \text{ â } D=\left\{(t,x):t>0,\ |x|\le l(t)=(n+1)\cdot\left(\frac{k}{kn-1}\right)^{\frac{n}{n+1}}[\tau(t)]^{\frac{1}{n+1}}\right\},$$

bu yerda u_{+} yuqorida aniqlangan funksiya, bu yechim bizga solishtirish teoremasi qo`llash imkonini beradi [10,11].

Endi $Lu_{+}\le 0$ ning D soha ekanligini ko'rsatamiz.

Aslida yuqori yechimni (2.15) ko`rinishida izlaymiz. Taxminan.

$$w(\tau,x)=\bar{f}(\xi),$$

(2.19) dan quyidagini hosil qilamiz

$$Lu_{+}\equiv-\frac{1}{n+1}\bar{f}-\tilde{u}_t\cdot\tilde{u}^{-kn}\cdot\tau(t)\cdot\bar{f}-\tilde{u}^{\beta-kn}\cdot\tau(t)\cdot\gamma(t)\cdot\bar{f}^{\beta}\ . \qquad (2.21)$$

$\bar{f}(\xi)$ funkciya (2.20) tenglamaning yechimi ekanligidan, $Lu_{+}\le 0$ shart bajarilishi uchun, quyidagi munosabatning bajarilishi yetarlidir

$$-\tau(t)\tilde{u}_t\tilde{u}^{-kn}\le\frac{1}{n+1}.$$

Bu esa tenglama shartiga ko’ra o’rinli.

$\bar{u}(t)$ ni tanlash hisobiga chegaraviy shart uchun quyidagi munosabatni olamiz $u(t,0)\le\psi(t)$. Bundan, barcha *t>0* lar uchun *u(t,x)* ning chegaralanganligi va (2.10),(2.11) masalaning global yechimga egaligi kelib chiqadi.

Теорема 2.2.1. isbotlandi.

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