BIR JINSLI DIFFERENSIAL TENGLAMAGA KELTIRILADIGAN DIFFERENSIAL TENGLAMALAR

Shomurodov Nozimbek Anvar o'g'li
nozimshoh007@gmail.com
Nurillayeva Gulchiroy Baxshullo qizi
Navoiy davlat konchilik va texnologiyalar universiteti, assistant

Biz ushbu tezisda bir jinsli tenglamaga keltiriladigan differensial tenglamalar haqida to`xtalib o`tamiz. Agar tenglamaning ko`rinishi quyidagicha bo`lsa,

Ushbu $\frac{dy}{dx}=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ tenglama (1)

1. $c_1=c_2=0$ bo'lganda bir jinsli diffrensial tenglama bo'ladi.

2. Agar c_1 va c_2 (yoki ulardan biri) noldan farqli bo'lsa, u holda (1) tenglama: a) $a_1b_2-a_2b_1\neq 0$ bo'lsa u holda $X=x-\alpha$, $Y=y-\beta$ almashtirishdan foydalanib $\frac{dY}{dX}=\frac{a_1X+b_1Y}{a_2X+b_2Y}$ yoki $\frac{dY}{dX}=\frac{a_1+b_1\frac{Y}{X}}{a_2+b_2\frac{Y}{X}}=\varphi\left(\frac{Y}{X}\right)$ X, Y larga nisbatan bir jinsli differensial tenglama hosil bo`ldi. Bu yerda α va β larni topish uchun $\begin{cases}a_1\alpha+b_1\beta+c_1=0\\a_2\alpha+b_2\beta+c_2=0\end{cases}$ sistemani yechamiz

b) Agar $a_1x+b_1y+c_1=0$ va $a_2x+b_2y+c_2=0$ chiziqlar o`zaro parallel bo`lsa, ya`ni $a_1b_2-a_2b_1=0$ bo`lsa, u holda $\frac{a_1}{a_2}=\frac{b_1}{b_2}=k$ deb belgilab olib, berilgan tenglamani $\frac{dy}{dx}=\frac{a_2kx+b_2ky+c_1}{a_2x+b_2y+c_2}=\frac{k(a_2x+b_2y)+c_1}{a_2x+b_2y+c_2}$ ko`rinishda yozish mumkin. Bu yerda $a_2x+b_2y=z$ almashtirish bajarib $\frac{dy}{dx}=\frac{kz+c_1}{z+c_2}$ tenglamaga ega bo'lamiz ikkinchi tomondan $z'=\frac{dz}{dx}=a_2+b_2\frac{dy}{dx}$ bundan $\frac{dz}{dx}=a_2+b_2\frac{kz+c_1}{z+c_2}$ o'zgaruvchilari ajraladigan differensial tenglama hosil qilamiz

Misol $y'=-\frac{2x+3y-1}{4x+6y-5}$ tenglamani umumiy yechimini toping?

Shartga ko'ra $a_1=2$, $b_1=3, c_1=-1, a_2=4, b_2=6, c_2=-5$ bundan $a_1b_2-a_2b_1=2\cdot 6-4\cdot 3=0$ shu sababli $2x+3y=z$ bundan $2+3y'=z'$ yoki $y'=\frac{z'-2}{3}$ u holda berilgan tenglama $\frac{z'-2}{3}=-\frac{z-1}{2z-5}$ bundan $z'=-\frac{3(z-1)}{2z-5}+2$ buni soddalashtirsak $z'=\frac{z-7}{2z-5}$ ko'rinishida bo'ladi $z'=\frac{dz}{dx}$ ekanligidan $\frac{dz}{dx}=\frac{z-7}{2z-5}$ o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz ya'ni $\frac{2z-5}{z-7}dz=dx$ bu tenglamani har ikkala tomonini integrallab $\int\frac{2z-5}{z-7}dz=\int dx+C$ (C=const) , $2z-9\ln|z-7|=x+C$ ga ega bo'lamiz z ni o'rniga belgilab olingan ifoda $2x+3y=z$ ni qo'yib $4x+6y-9\ln|2x+3y-7|=x+C$ umumiy yechimga ega bo'lamiz.

Xuddi shunday umumiy holda $\frac{dy}{dx}=f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$ ko'rinishdagi funksiyalar uchun xulosa chiqarish mumkin.

bu yerda $c_1^2 + c_2^2 > 0$ tenglama bir jinsli tenglamaga keladigan tenglamadir.

Koordinatalar boshini

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

agar, $a_1b_2 - a_2b_1 \neq 0$ chiziqlarning kesishgan nuqtasi (x_1, y_1) ga ko`chirsak, bu tenglama bir jinsli tenglamaga keladi. Ya`ni,

$$X = x - x_1, \quad Y = y - y_1$$

almashtirish bajaramiz. U holda

$$\frac{dY}{dX} = f\left(\frac{a_1X + b_1Y}{a_2X + b_2Y}\right)$$

yoki

$$\frac{dY}{dX} = f\left(\frac{a_1 + b_1\frac{Y}{X}}{a_2 + b_2\frac{Y}{X}}\right) = \varphi\left(\frac{Y}{X}\right)$$

X, Y larga nisbatan bir jinsli tenglama hosil bo`ldi. Agar $a_1x + b_1y + c_1 = 0$ va $a_2x + b_2y + c_2 = 0$ chiziqlar o`zaro parallel bo`lsa, ya`ni $a_1b_2 - a_2b_1 = 0$ bo`lsa, u holda $\frac{a_1}{a_2} = \frac{b_1}{b_2} = k$ deb belgilab olib, berilgan tenglamani

$$\frac{dy}{dx} = f\left(\frac{a_2kx + b_2ky + c_1}{a_2x + b_2y + c_2}\right) = f\left[\frac{k(a_2x + b_2y) + c_1}{a_2x + b_2y + c_2}\right] = F(a_2x + b_2y)$$

ko`rinishda yozish mumkin. Bu yerda $a_2x + b_2y = z$ almashtirish bajarib,

$$b_2\frac{dy}{dx} + a_2 = \frac{dz}{dx}$$

yoki

$$\frac{dz}{dx} = b_2F(z) + a_2$$

o`zgaruvchilari ajraladigan tenglamaga kelamiz. Ushbu differensial tenglamaga doir quyidagi misolni ko`rib o`tsak:

Misol: $(2x - 4y + 6)dx + (x + y - 3)dy = 0$ differensial tenglamani yeching.

Yechish: $(x + y - 3)dy = (4y - 2x - 6)dx$ differensial tenglamani, ushbu ko`rinishga keltirib olamiz:

$$\frac{dy}{dx} = \frac{4y - 2x - 6}{x + y - 3}$$

Bunda $\begin{cases} 4y - 2x - 6 = 0 \\ x + y - 3 = 0 \end{cases}$ ekanligidan, $x = 1$ va $y = 2$ kelib chiqadi.

$$x = X + 1 \qquad y = Y + 2$$

$\frac{dy}{dx} = \frac{4Y-2X}{X+Y}$, $z = \frac{Y}{X}$ belgilash kiritib, quyidagini hosil qilamiz:

$\frac{z+1}{3z-z^2-2}dz = \frac{1}{x}dx$ bundan

$$\ln\frac{(z-2)^3}{(z-1)^2} + \ln|x| = C$$

$$\ln\frac{(y-2x)^3}{(y-x-1)^2}=\ln C$$

$(y-2x)^3=C(y-x-1)^2$ ekanligi kelib chiqadi.

Ushbu tenglamalarga o`xshash oddiy differensial tenglamalarga oid bir nechta misollarni talabalar ixtiyoriga mustaqil o`rganib uni hayotga tadbiq etishlarini o`zlariga havola etamiz.

FOYDALANILGAN ADABIYOTLAR:

1. Sharipov Sh. R. va Mo`minov N.S. "Oddiy differensial tenglamalar" Toshkent. 1986 y.

2. Филиппов А. Ф "Сборник задач по дифференциальным уравнениям" Москва. "Наука" 1985 г.

3. Sh. R Xurramov “Oliy matematika masalalar to’plami nazorat topshiriqlari” 2-qism Toshkent 2015 y